Chapter 14. Expanding Universe

- *What does "expansion of the Universe"* mean *and how can I observe it?*
- *What does the Universe expand* from*? What does it expand* into*?*
- *How can a metric describe the Universe as a whole?*
- *You assume "for simplicity" that the Universe is uniform, but a glance at the night sky shows your assumption to be false!*
- *How many different kinds of uniform curvature are possible for the Universe as a whole?*
- *How do galaxies move as the Universe expands?*
- *How do we measure the distance to a remote galaxy?*
- *How far away "now" is the most distant galaxy that we see "now"?*
- *And what does "now" mean, anyway? Even special relativity shouts, "Simultaneity is relative!"*

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CHAPTER 14 Expanding Universe

Edmund Bertschinger & Edwin F. Taylor *

Nothing expands the mind like the expanding universe.

—Richard Dawkins

14.1 ■ DESCRIBING THE UNIVERSE AS A WHOLE

Finding words that correctly describe the unbounded

What is a one-sentence summary of our Universe? Try this:

One-sentence description of our Universe

> **Our visible Universe** consists of hundreds of billions of galaxies, each containing roughly one hundred billion stars, scattered more or less uniformly through a volume about 28 billion light years across.

A one-sentence description of *anything* is bound to be inadequate as a predictor of observed details; this and the following chapter expand(!) and correct this one-sentence description.

Assume a uniform Universe and that our location is not unique.

Figure 1 shows a small example of our visible Universe, which illustrates our assertion that galaxies are "scattered more or less uniformly." If so, this radically simplifies our model of the Universe: We describe the part we can see, and—in the absence of evidence to the contrary—assume the place we live is not unique but the same as any other location in the Universe. As a first—and it turns out, accurate—approximation, we look for metrics that describe curvature caused by a uniform distribution of mass. Make no assumption about how far this distribution extends. Instead, first, examine all possibilities consistent with general relativity; second, compute their predictions; third, let astronomical observations select the "correct" model or models.

Restrict attention to metrics that are uniform in space? Why not also uniform in time—a Universe that remains unchanged as the eons roll? In the absence of evidence to the contrary this would be the simplest hypothesis.

*Draft of Second Edition of *Exploring Black Holes: Introduction to General Relativity*

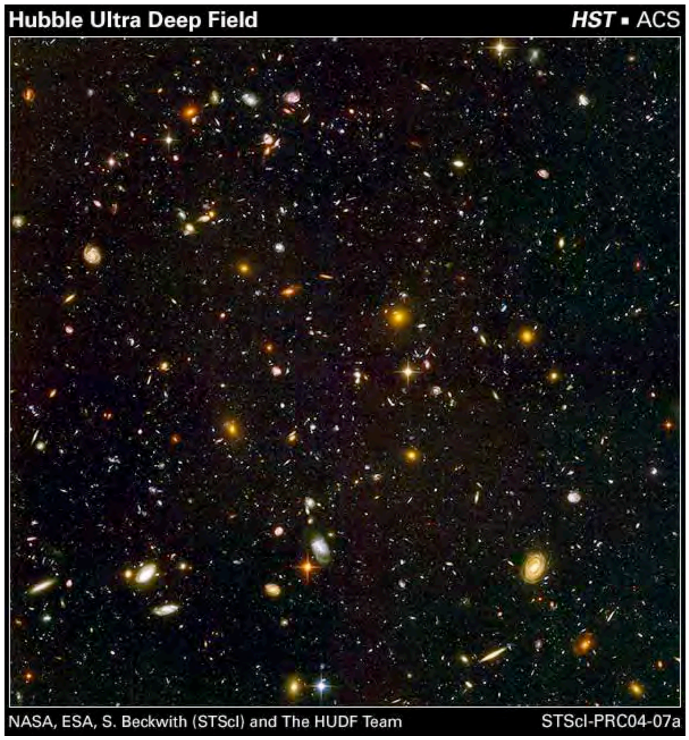


FIGURE 1 "Ultra deep field" image from the Hubble Space Telescope, named after astronomer Edwin P. Hubble. Every dot and every smear in this image is a galaxy, with the exception of a few nearby stars in our local galaxy. (Can you distinguish these exceptions?)

Cosmological constant comes, goes . . . then comes back again!

Indeed, in his 1917 cosmological model inspired by general relativity, Einstein looked for metrics that described a static Universe filled smoothly with mass. He found that no static metric was compatible with his newly-invented field equations unless he introduced a new term into those equations, a term that he called the **cosmological constant** and denoted by the Greek capital letter lambda, Λ. Later, after acknowledging Hubble's discovery that galaxies are flying away from one another, Einstein regretted the addition of Λ to his field equations. Astonishingly, today we know that there is something very similar, if not identical, to Λ at work in the Universe, as described in Chapter 15, Cosmology.

Brief history of the Universe

We know far more about the Universe than Einstein did a century ago. We know that the Universe is not static, but evolving. We know that approximately 14 billion years ago all matter/energy was concentrated in a much smaller structure. We know that this concentration expanded and thinned, from a moment we call the Big Bang, with galaxies forming during the initial expansion.

Box 1. Is this the only Universe?

Are there multiple universes, parallel universes, or baby universes? General relativity theorists write about all these and more. In this book we investigate the simplest model Universe consistent with observations—a single simply-connected spacetime.

Cosmologists often distinguish between "the observable universe" and all that there is or might be, citing plausible arguments that spacetime could be very different trillions of light years away. Here we restrict discussion to the simplest generalization of the observable universe, one—*the*—Universe that is everywhere similar to what we see in our vicinity.

Wait. Isn't science supposed to tell us what exists? Not at all! Science struggles to create theories that we can verify—or disprove—with observation and measurement.

How do we know?

How do we know these things? And how do we describe an evolving, expanding Universe? The present chapter assembles tools for this description, beginning with the metric of a spatially uniform, static Universe, then generalizes the metric to include general features of development with the t-coordinate. However, a detailed prediction of t-development requires a knowledge of the constituents of the Universe. Chapter 15, Cosmology provides this, then applies the tools assembled in the present chapter to analyze the past and predict alternative futures for our Universe.

14.2 ■ SPACE METRICS FOR A STATIC UNIVERSE

Describing a uniform space

Space metric for uniform space curvature

A Universe filled uniformly with mass and energy has—on average—*uniform space curvature* everywhere. In this book we deal mainly with two space dimensions plus a global t-coordinate. In one popular global map coordinate system, the most general constant-curvature *space* metric has the following form on the r, ϕ plane:

$$ds^2 = \frac{dr^2}{1 - Kr^2} + r^2 d\phi^2 \tag{1}$$

Flat, closed, and open spaces

The value of the parameter K determines the shape of the space, which in turn determines the range of r:

$$\text{for } K = 0, \qquad 0 \leq r < \infty \qquad \text{(Case I: flat space)} \tag{2}$$

$$\text{for } K > 0, \qquad 0 \leq r \leq \frac{1}{K^{1/2}} \qquad \text{(Case II: closed space)} \tag{3}$$

$$\text{for } K < 0, \qquad 0 \leq r < \infty \qquad \text{(Case III: open space)} \tag{4}$$

Flat plane, sphere, and saddle

Preview: We easily visualize Case I, flat space—equation (2). Next we visualize Case II, closed space, as a sphere—equation (3) and Figure 2. Finally Case III, open space has the shape of a saddle—equation (4) and Figure 3.

To describe the expansion of the Universe, it is helpful to separate its scale or size, symbolized by a **scale factor** R, from its curvature described by a space metric that uses the unitless coordinate χ ("chi," rhymes with "high"), the lower-case Greek letter that corresponds to the Roman x.

Traveling in flat space

Case I: flat space. For flat space, equation (2) tells us that $K = 0$ in (1). For this case the r-coordinate is simply the product of the scale factor R and the unitless coordinate χ:

$$r = R\chi \qquad \text{so that} \qquad dr = R d\chi \qquad (\text{flat space}, \; 0 \le \chi < \infty) \tag{5}$$

This leads to the metric for flat space:

$$ds^2 = R^2\left(d\chi^2 + \chi^2 d\phi^2\right) \qquad (\text{flat space}, \; K = 0 \text{ and } 0 \le \chi < \infty) \tag{6}$$

If you start walking "straight in the χ-direction" in a flat space, you do not return to your starting point.

Variable χ automatically satisfies limits.

Case II: closed space. Limits on the r-coordinate in (3) for a closed space can be automatically satisfied with a coordinate transformation. Let

$$r \equiv \frac{1}{K^{1/2}} \sin\chi \qquad (K > 0 \text{ and } 0 \le \chi \le \pi) \tag{7}$$

The sine function automatically limits the range of r to that given in (3). The coordinate r is a troublemaker; it has the same value in the two hemispheres of the sphere (Figure 2). But we use the coordinate χ, which does not have this problem; it is single-valued.

The differential dr is

$$dr = \frac{1}{K^{1/2}} \cos\chi d\chi \qquad (K > 0 \text{ and } 0 \le \chi \le \pi) \tag{8}$$

With these transformations the metric for the closed, constant-curvature space (1) and (3) becomes

$$ds^2 = \frac{1}{K}\left(d\chi^2 + \sin^2\chi \, d\phi^2\right) \qquad (\text{closed space}, \; K > 0 \text{ and } 0 \le \chi \le \pi) \tag{9}$$

Equation (9) is equivalent to the space metric for the surface of Earth, equation (3), Section 2.3:

$$ds^2 = R^2(d\lambda^2 + \cos^2\lambda \, d\phi^2) \qquad (\textit{space metric}: \text{Earth's surface}) \tag{10}$$

Expressions in parentheses on the right sides of both (9) and (10) refer to the unit sphere. In Chapter 2 we used the latitude λ rather than the colatitude χ. The two are related by the following equation, illustrated in Figure 2:

$$\chi \equiv \frac{\pi}{2} - \lambda \tag{11}$$

Transformation (11) replaces the sine in (9) with the cosine in (10).

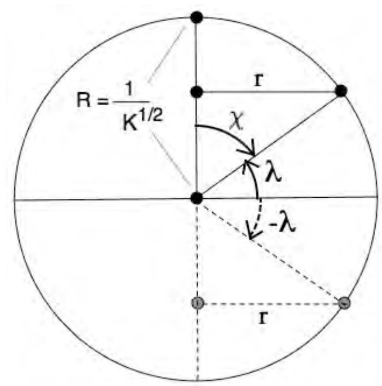


FIGURE 2 Relation between latitude λ and colatitude χ to determine the north-south coordinate on the sphere with $R = 1/K^{1/2}$ in Euclidean space. Latitude λ ranges over the values $-\pi/2 \leq \lambda \leq +\pi/2$, whereas colatitude χ ranges over $0 \leq \chi \leq \pi$. Equation (11) gives the relation between χ and λ, while (7) gives the relation between χ and r. This figure also shows that r is a "bad" coordinate, since it is double-valued, failing to distinguish between northern and southern latitude. In contrast, χ is single-valued from $\chi = 0$ (north pole) to $\chi = \pi$ (south pole).

Describing closed space

Thus for $K > 0$ the shape of constant-curvature space is that of a spherical surface with a scale factor R whose square is equal to $1/K$. The space represented by the surface of the sphere is homogeneous and isotropic: the same everywhere and in all directions. Same shape in this model means same physical experience in its predictions. In addition, if you start walking "straight in the χ-direction" in this closed space, you return eventually to your starting point.

When we use R instead of K, equation (9) becomes

$$ds^2 = R^2 \left(d\chi^2 + \sin^2\chi\, d\phi^2\right) \qquad \text{(closed space, } 0 \leq \chi \leq \pi) \qquad (12)$$

where the expression in the parenthesis on the right side also embodies the shape of the unit sphere.

Comment 1. Scale factor R?
In Figure 2, R is the radius of a sphere in Euclidean space. In equation (12) R is a scale factor in curved spacetime. Euclid does not describe curved spacetime, so what does "scale factor" mean for the description of our Universe? We cannot answer this question until we know what the Universe contains, the subject of the following chapter. In the meantime we continue to play the dangerous analogy between points in flat space and events in curved spacetime begun in Chapter 2.

Describing open space

Case III: open space. Values $K < 0$ in metric (1) lead to an *open* space, as shown by the alternative transformation:

$$r \equiv R \sinh\chi \qquad \text{(open space, } 0 \leq \chi < \infty) \qquad (13)$$

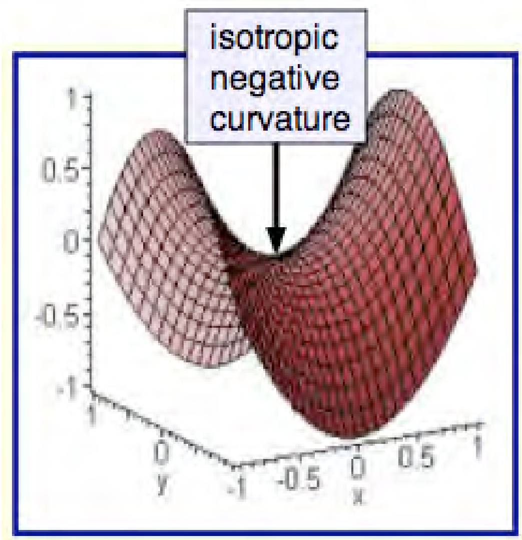


FIGURE 3 The saddle shape has intrinsic negative curvature. Only in the neighborhood of a single (central) point, however, is the negative curvature the same in all directions. Elsewhere on the surface the curvature is negative but varies from place to place and is different in different directions. (It is mathematically impossible to embed in three spatial dimensions a two-dimensional surface that has uniform negative curvature everywhere.)

where $R^2 = -1/K$ and sinh is the hyperbolic sine. The hyperbolic sine and cosine are defined by the equations

$$\sinh\chi \equiv \frac{e^{\chi} - e^{-\chi}}{2} \qquad \text{and} \qquad \cosh\chi \equiv \frac{e^{\chi} + e^{-\chi}}{2} \tag{14}$$

Equation (13) shows r to be a monotonically increasing function of χ, so there is no worry about a single value of r representing more than one location. The differential dr is

$$dr = R\cosh\chi\, d\chi \qquad (\text{open space},\ \ 0 \leq \chi < \infty) \tag{15}$$

and the corresponding space metric is

$$ds^2 = R^2\left(d\chi^2 + \sinh^2\chi d\phi^2\right) \qquad (\text{open space, } K < 0 \text{ and } 0 \leq \chi < \infty) \tag{16}$$

The expression in the parentheses on the right side of this equation embodies an open space that has a uniform negative curvature. The saddle surface shown in Figure 3 has a single central point whose curvature is negative and the same in all directions. That is the *only* point on the surface with the same curvature in all directions. Unfortunately it is not possible to embed in three spatial dimensions a two-dimensional surface that has uniform negative

Box 2. What does the Universe expand into?

A common misconception is that the Universe expands in the same way that a balloon expands or a firecracker explodes: into a pre-existing three-dimensional space. That is wrong: Spacetime comes into existence with the Big Bang and develops with t.

If you stick with the image of the expanding balloon for the closed Universe, the model correctly requires you to assume that the surface of the balloon is all that exists. Galaxies are scattered across its surface and human observers are surface creatures who view nothing but what lies on that surface. At the beginning of expansion, the surface evolves from a point-event that is also the beginning of time—the so-called **Big Bang**. During the subsequent expansion, every surface creature sees other points on the balloon move away from him, and points farther from him move away faster. In this model, the balloon does not expand *into* space, it represents *all* of space.

curvature everywhere. The best we can do is the saddle shape, with its single point of isotropic negative curvature.

14.3 ■ ROBERTSON-WALKER GLOBAL METRIC

A Universe that expands

"Expands" means $R(\text{constant}) \rightarrow R(t)$

We hear that the Universe "expands with time." What does that mean? Space metric (12) describes the surface of Earth, with R equal to Earth's radius. Suppose we inflate the Earth like a balloon. Then R increases with t while its property of uniform space curvature remains. By analogy, to describe a Universe that expands while keeping the same shape, we replace the static scale factor R in equations (12), (16), and (6) with a scale factor $R(t)$ that increases with t. In the 1930s, Howard Percy Robertson and Arthur Geoffrey Walker proved that the *only* spacetime metric that describes an evolving, spatially uniform Universe takes the form:

$$d\tau^2 = dt^2 - R^2(t)\left[d\chi^2 + S^2(\chi)d\phi^2\right] \qquad \text{(Robertson-Walker metric)} \qquad (17)$$

Robertson-Walker metric

To describe different shapes of the Universe, we modify the function $S(\chi)$ by generalizing equations (5), (7), and (13) respectively:

$$S(\chi) = \chi \qquad (\text{flat Universe}, \quad 0 \leq \chi < \infty) \qquad (18)$$

$$S(\chi) = \sin\chi \qquad (\text{closed Universe}, \quad 0 \leq \chi \leq \pi) \qquad (19)$$

$$S(\chi) = \sinh\chi \qquad (\text{open Universe}, \quad 0 \leq \chi < \infty) \qquad (20)$$

Comoving coordinates

Coordinates χ and ϕ are called **comoving coordinates** because a galaxy with fixed χ and ϕ simply "rides along" as the scale function $R(t)$ increases.

For a closed Universe, $R(t)$ might be interpreted loosely as the "radius of the Universe." However, for flat or open Universes, $R(t)$ has no such simple interpretation. We simply call R the **scale function of the Universe.**

Box 3. Is a static, uniform Universe possible?

The Robertson-Walker metric (17) is more general than general relativity. Whether or not the Robertson-Walker metric satisfies Einstein's field equations depends on variation of the scale function $R(t)$ with the global t coordinate. At any value of t, the function $R(t)$ depends on what the Universe is made of and how much of each constituent is present at that t and was present at smaller t. Chapter 15, Cosmology, examines the presence and density of the constituents of the Universe at different global t-coordinates, then displays the resulting functions $R(t)$ that satisfy Einstein's equations, and finally traces the consequences for our current model of the development of the Universe. In the present chapter we simply assume that $R(t)$ starts with value zero at the Big Bang and thereafter increases monotonically.

In 1917 Einstein thought that the Universe was not only uniform in space, but also unchanging in t. Such a spacetime has the spacetime metric (17) with R a constant. Is this a valid metric for the Universe?

Einstein showed that metric (17) with R = constant does not satisfy his field equations for a Universe uniformly filled with matter. However, by adding the cosmological constant Λ to his field equations, he obtained a unique solution for a closed Universe, the case described by (19). The effect of Λ is to create a cosmic repulsion that keeps galaxies from being drawn together by gravity. Chapter 15, Cosmology, shows that something very much like Λ—now called $dark\ energy$—repels galaxies, so at the present stage of the Universe distant galaxies fly away from our own galaxy with increasing speed.

YOU ARE AT THE "CENTER OF THE UNIVERSE."

For all three models of the Universe described by (18) through (20), the location $\chi = 0$ appears to be a favored point, for example the north pole for the closed Universe or the center of the saddle for the open Universe or an origin anywhere in the flat Universe. Because the Universe is assumed to be completely uniform, however, we can choose *any* point as $\chi = 0$ (and as the origin of ϕ). That arbitrary point then becomes the north pole or the center of the saddle or the origin in flat space. The mathematical model permits every observer to assume that s/he is at the center of the Universe. (Talk about ego!)

Global t on wristwatch of comoving observer

The squared t-differential dt^2 in (17) has the coefficient one; in Robertson-Walker map coordinates, t has no warpage. Indeed, for $d\chi = d\phi = 0$, passage of coordinate t tracks the passage of wristwatch time τ. The interpretation is simple: coordinate t is that recorded on comoving clocks, those that ride along "at rest" with respect to the space coordinates of the expanding Universe.

Space and time exist only for $t > 0$.

We should also give a range for coordinate t in order to complete the definition of the spacetime region described by equations (17) through (20). However we cannot specify a range of t until we know details of the scale function $R(t)$. For Big Bang models of the Universe—expansion from an initial singularity—the scale function starts with $R(t) = 0$ at $t = 0$. In this book we examine Big Bang models, for which spacetime exists only for $t > 0$.

14.4 ■ REDSHIFT

Light we receive from far away increases in wavelength in an expanding Universe.

Choose the center of the Universe to be at *my location*, and t_0 to be *now*.

We are free to choose the center of the Universe at our location, that is at $\chi = 0$ and to assume that we stay at the center permanently. Then every current observation that we make is an event that takes place at $\chi = 0$ and *now*, which we will call $t = t_0$.

$$\text{Observation NOW on Earth has map coordinates } t \equiv t_0,\ \chi \equiv 0 \qquad (21)$$

Suppose that a distant star is fixed in comoving coordinates χ and ϕ, so it rides along as the scale function $R(t)$ increases. Let the star emit a light flash at $(t_{\rm emit}, \chi_{\rm emit})$, which we observe on Earth at $(t_0, 0)$.

For light, $d\tau = 0$ and for radial motion $d\phi = 0$ in metric (17). Write the resulting metric with t and space terms on opposite sides of the equation, take the square root of both sides, and integrate each one:

$$\int_{t_{\rm emit}}^{t_0} \frac{dt}{R(t)} = \int_0^{\chi_{\rm emit}} d\chi = \chi_{\rm emit} \qquad (\text{light, d}\phi = 0) \qquad (22)$$

Emit and detect two light flashes.

Think of a second light flash emitted from the same star at event $(t_{\rm emit} + \Delta t_{\rm emit}, \chi_{\rm emit})$ and observed by us at $(t_0 + \Delta t_0, 0)$. The two flashes can represent two sequential positive peaks in a continuous wave. We assume that the emitter is located at constant χ, so the second flash travels the same χ-coordinate difference as the first. Hence the right-hand integral has the same value for both flashes. Therefore

$$\int_{t_{\rm emit}+\Delta t_{\rm emit}}^{t_0+\Delta t_0} \frac{dt}{R(t)} = \chi_{\rm emit} \qquad (\text{light}) \qquad (23)$$

Compare the t-limits of the integrals on the left sides of (22) and (23). The integration in (23) starts later by $\Delta t_{\rm emit}$ and ends later by Δt_0. In consequence, when we subtract the two sides of equation (22) from the corresponding sides of equation (23), the result is:

$$\int_{t_0}^{t_0+\Delta t_0} \frac{dt}{R(t)} - \int_{t_{\rm emit}}^{t_{\rm emit}+\Delta t_{\rm emit}} \frac{dt}{R(t)} = 0 \qquad (\text{light}) \qquad (24)$$

Approximate this equation to first order in $\Delta t_{\rm emit}$ and Δt_0, leading to

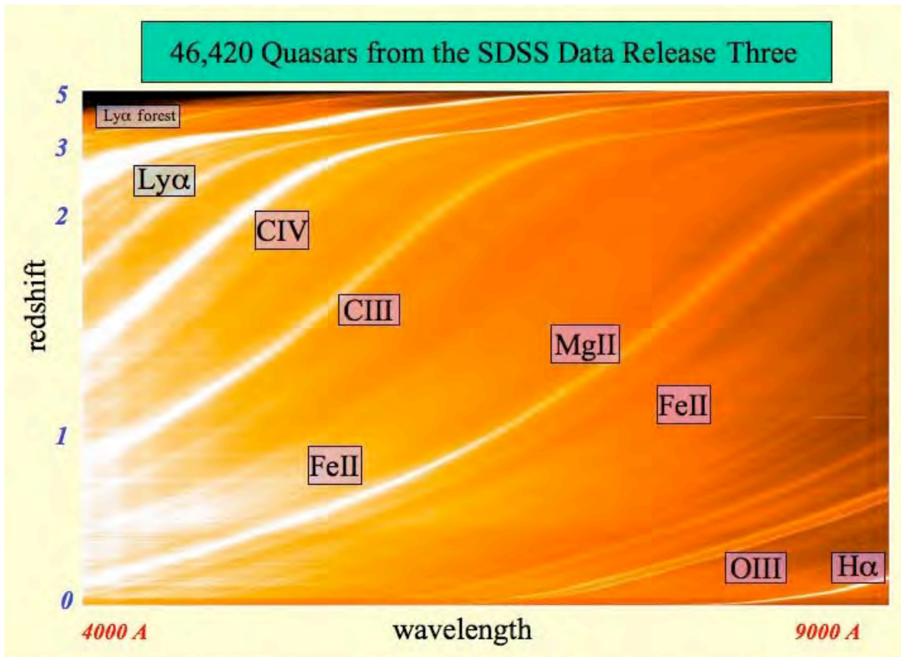
$$\frac{\Delta t_0}{R(t_0)} \approx \frac{\Delta t_{\rm emit}}{R(t_{\rm emit})} \qquad (\text{light}) \qquad (25)$$

Let the two flashes represent two sequential peaks in a continuous wave. Then the lapse in t between flashes in meters that each observer measures equals the wavelength in meters.

$$\frac{\Delta t_0}{\Delta t_{\rm emit}} = \frac{\lambda_0}{\lambda_{\rm emit}} = \frac{R(t_0)}{R(t_{\rm emit})} \qquad (\text{light}) \qquad (26)$$

Redshift z

In this equation an equality sign replaces the approximately equal sign in (25) because one wavelength of light λ is truly infinitesimal compared with the scale function $R(t)$ of the Universe. It is customary to measure the fractional



In addition to images, the SDSS has measured the spectra of light from more than a million celestial sources. The spectrum of an object shows the intensity of its light as a function of wavelength. This picture shows the spectra of more than 46,000 quasars from the SDSS 3rd data release; each spectrum has been converted to a single horizontal line, and they are stacked one above the other with the closest quasars at the bottom and the most distant quasars at the top. Bright bands show the emission produced by specific ions of hydrogen, carbon, oxygen, magnesium, and iron. For more distant quasars, these emission lines are shifted to longer wavelengths by the expansion of the universe. This redshift of spectral lines is what the SDSS measures to determine the distances to quasars and galaxies.
Credit: X. Fan and the Sloan Digital Sky Survey.

FIGURE 4 A remarkable plot of the redshifts z of the spectra from more than 46 thousand quasars taken by the Sloan Digital Sky Survey (SDSS). The spectrum of each quasar lies along a single horizontal line at a vertical position corresponding to its redshift z. Some prominent spectral lines from different atoms are labeled: Lyα is the Lyman alpha line of hydrogen. Roman numeral I following an element is the neutral atom; Roman numeral II is the singly ionized atom, and so forth. Thus MgII is singly ionized magnesium and CIV is triply ionized carbon. The observed wavelength λ_0 increases with increasing z. (The redshift scale is nonlinear so the bands are not straight lines.)

change in wavelength using a dimensionless parameter z, called the **redshift**, defined by the equation

$$\lambda_0 \equiv (1+z)\lambda_{\rm emit} \qquad \text{(light)} \qquad (27)$$

Stretch factor: $1+z$

where we call $1+z$ the **stretch factor**. Then equation (26) can be written

$$1+z \equiv \frac{\lambda_0}{\lambda_{\rm emit}} = \frac{R(t_0)}{R(t_{\rm emit})} \qquad \text{(stretch factor)} \qquad (28)$$

In other words, when we train our telescopes on a source with redshift z, we observe light emitted at the t-coordinate when the Universe scale function $R(t)$ was a factor $1/(1+z)$ the size it is today.

Cosmological redshift

The change in wavelength described by equation (28) is called the **cosmological redshift**. The observation t_0 is greater than the emission t_{emit}, and for an expanding universe $R(t_0) > R(t_{\text{emit}})$. Therefore the observed light has a longer wavelength than the emitted light; the color of light visible to our eyes shifts toward the red end of the spectrum, hence the term "redshift." The same fractional increase in wavelength occurs for electromagnetic radiation of any frequency, so the term *redshift* applies to microwaves, infrared, ultraviolet, x-rays, and gamma rays.

Redshift a Doppler shift?

Equation (27) appears not to describe a Doppler shift in the special relativity sense. Both emitter and observer are *at rest* in their comoving coordinate χ; nevertheless, they observe the light to have different wavelengths. In a sense the expansion of the Universe "stretches out" the wavelength of the light as it propagates. In another sense, however, the cosmological redshift is a cumulative redshift, because a star at fixed χ is at an $R(t)\chi$ that grows with t. In other words, it moves away from us. Section 14.7 shows that for $z \ll 1$, the cosmological redshift *is* a Doppler shift.

Redshift deduced from laboratory spectra

When we see light of a given frequency that has been emitted from a distant galaxy, how do we know that it has been redshifted? With what do we compare it? From laboratory experiments on Earth, we know the discrete spectrum of radiation frequencies emitted by a particular atom or molecule. Then the identical *ratios* of frequencies of light received from a distant star tell us what element or molecule we are observing in that star. And from the value of the shift at any one frequency we can deduce the redshift for all frequencies. Figure 4 shows redshifted spectral lines (bright: emission lines; dark: absorption lines) of light from many different atoms in distant quasars.

Astronomers use z for t_{emit}.

Because it is easy to measure a galaxy's redshift z, astronomers use z as a proxy for t_{emit} in equation (26)—Figure 5. Whenever you read a news article about a galaxy formed during the first billion years of the Universe, remember that astronomers do not measure t; they measure redshift. The distant galaxies in the news have $z > 6$: in the process of traveling to us, the wavelength of their light has been stretched by a factor more than 7! Light in our visual spectrum has been redshifted to the infrared. This is why the James Webb Space Telescope—the successor to the Hubble Space Telescope—looks in the infrared region of the spectrum for light from the most distant galaxies, those that appeared earliest in the evolution of the Universe.

14.5 ■ HOW DO GALAXIES MOVE?

Apply the Principle of Maximal Aging to the motion of a galaxy.

Transverse galaxy motion is difficult to detect.

We have a disability in viewing the distant Universe: we are limited to effectively a single point, the Earth and its solar system. The redshift of light from distant galaxies gives us a handle on their radial recession. However,

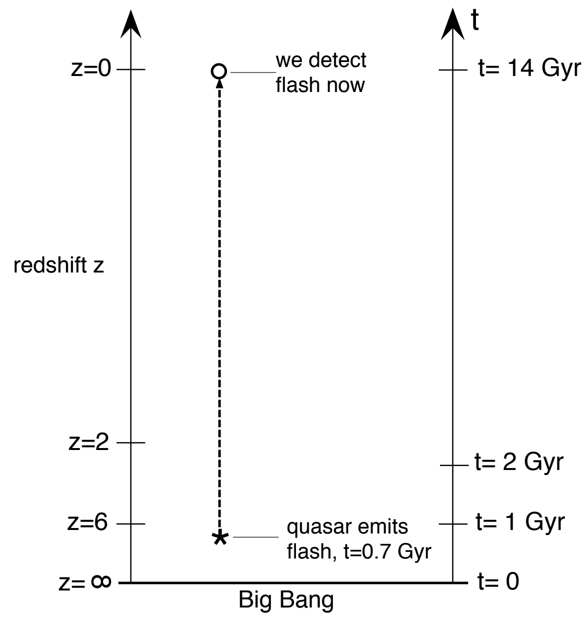


FIGURE 5 Schematic diagram comparing redshift z with cosmic t, in units of Gigayears (10^9 years). Calibration of the scale at the right of the figure depends on the t-development of the Universe, through $R(t)$, based on our current model. Astronomers use redshift as a proxy for t, both because it is directly measurable and also because it does not change as we revise our scale of cosmic t. The flash emission and detection is the case analyzed in Box 4.

Limit attention to radial motion.

transverse motion of a remote source is too small to detect directly in a human lifetime. (See the exercises.) In this and following sections, however, we limit attention to sources that move radially away from us.

Galaxy motion from Principle of Maximal Aging

How do galaxies move in the global coordinate system of metric (17)? As usual, the metric tells us about the structure of spacetime but does not determine the motion of a stone—or a galaxy. For that we need the Principle of Maximal Aging, which requires that total wristwatch time be a maximum along the worldline of a free galaxy that crosses adjoining flat patches.

For radial motion, the metric (17) becomes:

$$d\tau^2 = dt^2 - R^2(t)d\chi^2 \qquad (d\phi = 0) \tag{31}$$

Seek a conserved quantity.

This metric is valid for any function $S(\chi)$ in (17), whether for a flat, closed, or open model Universe. By just looking at this metric, can we anticipate constants of motion? One metric coefficient depends explicitly on t through the function $R(t)$. All our earlier derivations of map energy as a constant of motion required that no metric coefficient be an explicit function of t. Therefore metric (17) tells us that energy will *not* be conserved in the motion of galaxies. However, for radial motion ($d\phi = 0$) the metric coefficients do not

Box 4. How far away (now) is the most distant galaxy that we see (now)?

We see *now* the most distant galaxies as they *were* when they emitted the light: at, say, $t_{\rm emit} = 0.7$ billion years after the Big Bang (Figure 5). The current age of the Universe is $t_0 \approx 14$ billion years, so $t_0 - t_{\rm emit} \approx 13.3$ billion years. Naively, then, we might expect that these galaxies lie about 13 billion light years from us. However, this is false; they must lie much further away at the present day. Why? Because these galaxies have moved farther away from us during the 13.3 billion years that it took for their light to reach us. How much farther? What is the "true" map distance *now* between us and a galaxy formed at $t_{\rm emit} = 0.7$ billion years ago? In this case the word "true" has meaning only through the metric.

Use the Robertson-Walker metric (17) with $d\tau = 0$ to obtain the map distance between the emitting galaxy (at $\chi = \chi_{\rm emit}$) and Earth (at $\chi = 0$) at any particular t. This map distance is given simply by $R(t)\chi_{\rm emit}$, since the emitter continually "rides along" at the constant comoving coordinate $\chi_{\rm emit}$. The present separation $d_0 \equiv \sigma_0$ is then just $R(t_0)\chi_{\rm emit}$ with $\chi_{\rm emit}$ given by (22).

$$d_0 = R(t_0)\chi_{\rm emit} = R(t_0)\int_{t_{\rm emit}}^{t_0} \frac{dt}{R(t)} \qquad (29)$$

We cannot complete this calculation until we know how the scale function $R(t)$ increases with t. That is the task of Chapter 15. For a rough estimate of the present map distance d_0, assume that the scale function increases uniformly with t: $R(t)/R(t_0) = t/t_0$. Then the integral in (29) can be carried out using $t_{\rm emit} = 0.7$ billion years and the present $t_0 = 14$ billion years:

$$\begin{aligned} d_0 &= t_0 \int_{t_{\rm emit}}^{t_0} \frac{dt}{t} = t_0 \ln\frac{t_0}{t_{\rm emit}} \qquad (30)\\ &= t_0 \ln\frac{14}{0.7} = 14 \times 3.0 = 42 \end{aligned}$$

in billions of light-years. We call d_0 the **look-back distance**. According to this rough model, look-back distances of galaxies that emitted light 13 billion years ago are something like $d_0 = 42$ billion light years. This is their calculated map distance away from us now. We can refine this estimate by using a more accurate scale function $R(t)$; the present look-back distance to these remote galaxies is almost certainly larger than 42 billion light years.

depend explicitly on χ, so there will be a conserved quantity related to motion in χ, a kind of radial momentum.

The galaxy crosses two adjoining patches (Figure 6). Label A and B the segments of its path across the respective patches. Consider three events: Two at the opposite edges of the patches and one where they join. To find momentum as a constant of motion, we fix the t of all three events and fix the locations of the two events at the outer ends of the two segments. Then we vary the χ-coordinate of the connecting event (and the boundary between patches) in order to maximize total wristwatch time.

Over one patch, $R(t)$ is treated as being constant, so each patch is flat. Define

$$R_{\rm A} \equiv R(\bar{t}_{\rm A}) \qquad \text{and} \qquad R_{\rm B} \equiv R(\bar{t}_{\rm B}) \qquad (32)$$

where $\bar{t}_{\rm A}$ and $\bar{t}_{\rm B}$ are the average t-values when the galaxy crosses patch A and B, respectively. Define t for the galaxy to cross each patch as:

$$\begin{aligned} t_{\rm A} &\equiv t_{\rm middle} - t_{\rm start} \qquad (33)\\ t_{\rm B} &\equiv t_{\rm end} - t_{\rm middle} \end{aligned}$$

Let $\chi_{\rm A}$ be the *change* in coordinate χ across segment A and $\chi_{\rm B}$ be the corresponding change across segment B. Then $R_{\rm A}\chi_{\rm A}$ is the radial separation

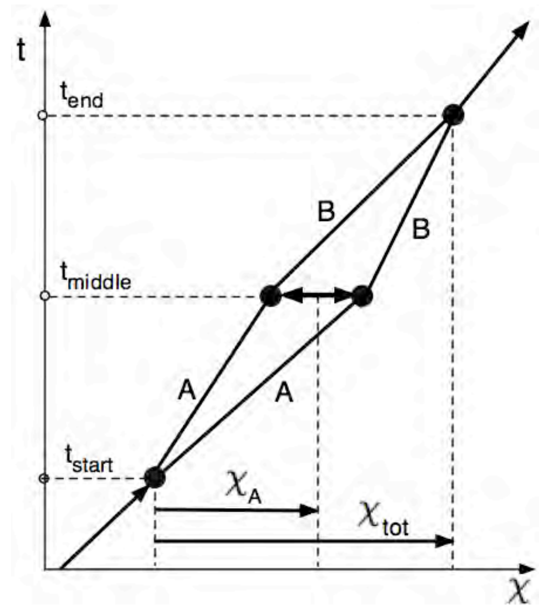


FIGURE 6 Greatly magnified picture of alternative worldlines across incremental segments A and B used in the derivation of the constant of motion (38). We vary the position χ_A of the middle event between segments A and B and demand that the total wristwatch time across both segments be maximum. The origin of this diagram is NOT necessarily at the zero of either t or radial position.

across segment A and $R_B(\chi_{tot} - \chi_A)$ the radial separation across segment B, with χ_A variable. Then the metric (31) across the two patches becomes:

$$\tau_A = \left[t_A^2 - R_A^2\,\chi_A^2\right]^{1/2} \tag{34}$$

and

$$\tau_B = \left[t_B^2 - R_B^2\,(\chi_{tot} - \chi_A)^2\right]^{1/2} \tag{35}$$

Fix t_{start}, t_{middle}, and t_{end} at the edges of the two segments. This fixes the values of t_A, t_B, R_A, and R_B through equations (32) through (35).

Now vary χ_A to maximize the total wristwatch time $\tau_{tot} = \tau_A + \tau_B$ across both segments:

$$\begin{aligned}\frac{d\tau_{tot}}{d\chi_A} &= \frac{d\tau_A}{d\chi_A} + \frac{d\tau_B}{d\chi_A} \\ &= -\frac{R_A^2\,\chi_A}{\tau_A} + \frac{R_B^2\,(\chi_{tot} - \chi_A)}{\tau_B} \\ &= -\frac{R_A^2\,\chi_A}{\tau_A} + \frac{R_B^2\,\chi_B}{\tau_B} = 0\end{aligned} \tag{36}$$

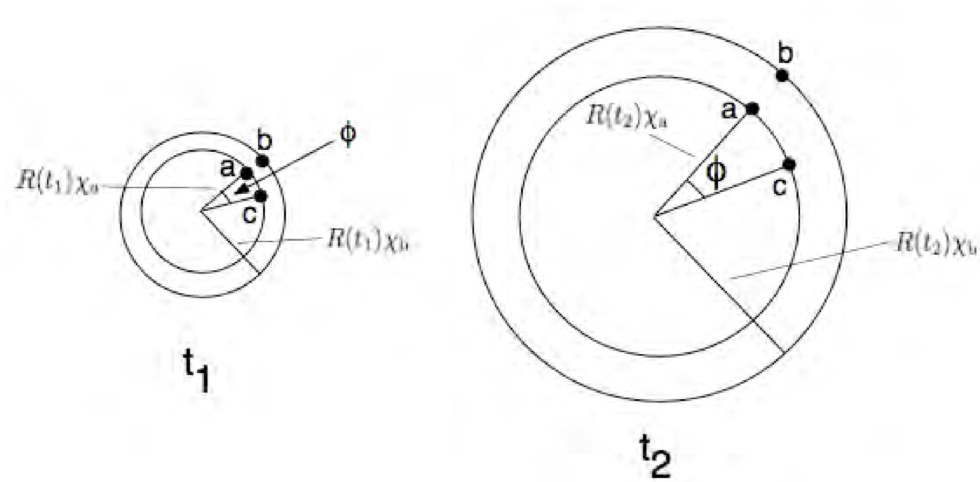


FIGURE 7 One possible radial motion for a galaxy is to remain at rest in the comoving coordinate χ and ϕ and ride outward, following $R(t)$, as the Universe expands. This figure shows the result for a flat Universe. All separations increase by the same ratio, so every observer can analyze galaxy motion with himself at the center and galaxies expanding away from him.

or

$$\frac{R_{\mathrm{B}}^2\,\chi_{\mathrm{B}}}{\tau_{\mathrm{B}}} = \frac{R_{\mathrm{A}}^2\,\chi_{\mathrm{A}}}{\tau_{\mathrm{A}}} \tag{37}$$

Now the usual argument: The left side of (37) refers to parameters of segment B alone, the right side to parameters of segment A alone. We have found a quantity that has the same value for each segment—that is, a constant of motion. Restore differentials and define a constant of motion Q_{r}.

Constant Q_{r} for radial motion only

$$Q_{\mathrm{r}} \equiv mR^2\frac{d\chi}{d\tau} = R\left(\frac{mRd\chi}{d\tau}\right) \equiv Rp_{\mathrm{r}} \quad \text{is a constant of motion} \tag{38}$$

where (38) provides a definition of local radial momentum p_{r} because $Rd\chi$ is a measured distance, from (17). Here m is the mass of a stone—or of a galaxy! Let the motion be radial only, so $p_{\mathrm{r}} = p$. Then (38) is still valid as $m \rightarrow 0$ for a photon, with $p = E$. In other words $R(t)E$ is constant for light, which means that as $R(t)$ increases, the energy E of photons decreases—another example of cosmological redshift.

Constant of motion for galaxy or light

We can distinguish two possible radial motions of a galaxy that leave Q_{r} constant. In the first, χ remains constant as t increases, so $d\chi/d\tau = 0$ and $Q_{\mathrm{r}} = p_{\mathrm{r}} = 0$. Each such "comoving" galaxy rides outward with $R(t)$; two galaxies at different values of χ move apart as $R(t)$ increases with t. For flat space ($S = \chi$) one can think of a set of concentric rings of galaxies fixed in the

Two possible radial motions

comoving coordinate χ. As t increases, the radius of each ring increases with $R(t)$. Figure 7 shows that radial separations $R(t)\chi$ and tangential separations $R(t)\chi\phi$ both increase proportionally to $R(t)$. This is true for every observer. There is no unique center; every observer can plot the expansion of the Universe in global coordinates with himself at the center.

In the second possible radial motion that leaves $Q_{\rm r}$ constant, a galaxy moves radially with respect to comoving coordinate χ. (Most galaxies have at least a slightly non-zero $Q_{\rm r}$ because of local gravity from spatial inhomogeneities.) Or one can think of a stone thrown radially out of a comoving galaxy. For such motion one can rewrite (38) as:

$$p_{\rm r} = \frac{Q_{\rm r}}{R(t)} \tag{39}$$

Q_r remains constant and $R(t)$ increases, so $p_{\rm r}$ decreases. This is called the "cosmological redshift of momentum." The high speed limit on (39) applies to a photon:

$$E = p \propto \frac{1}{R(t)} \qquad \text{(light)} \tag{40}$$

Constant Q_ϕ for *any* motion

We can derive another constant of motion, one that is valid for *any* free motion in Robertson-Walker global coordinates. Apply the Principle of Maximal Aging to two patches separated in ϕ-coordinate instead of χ-coordinate. The result is

$$Q_\phi \equiv mR^2S^2\frac{d\phi}{d\tau} = RS\left(\frac{mRSd\phi}{d\tau}\right) \equiv RSp_\phi \tag{41}$$

$$\text{(constant for } \textit{any} \text{ free motion)}$$

Equation (41) provides a definition of local tangential momentum p_ϕ because $RSd\phi$ is a measured distance, from metric (17).

14.6 ■ MEASURING DISTANCE

Extending a ruler from one lonely outpost.

So much for the theory of how galaxies move in the expanding Universe. What predictions does theory make about observations? On Earth we describe motion by plotting distance vs. time. Life in the Universe is more complicated. There are two problems: We cannot directly measure distances to objects outside our galaxy, and we cannot directly measure times longer than a few centuries. What hope can we have, therefore, to measure billions of years and billions of light years in the Universe?

Problems with our observations

First we give up trying to measure time. Instead we measure distance and velocity, both through indirect means. Section 14.7 discusses velocity measurements through redshift of spectral lines; here we focus on distance.

Box 5. Edwin P. Hubble

FIGURE 8 Edwin P. Hubble on the cover of Time Magazine, 1948.

Edwin P. Hubble was as important to astronomy as Copernicus. He expanded our view of the Universe from a single home galaxy to many galaxies that are rushing away from one another.

Hubble was born in 1889. In his youth he was an outstanding athlete and one of the first Rhodes Scholars at Oxford University, England. After returning to the United States he taught Spanish, physics, and mathematics in high school. He served in World War I, after which he earned a Ph.D. at the Yerkes Observatory of the University of Chicago.

In 1919 Hubble took up a position at Mount Wilson Observatory where he used the new 100-inch Hooker reflecting telescope, with which he discovered and analyzed redshifts of light from what were called “nebulae.” At that time the prevailing view was that the Universe consisted entirely of our galaxy. Hubble showed that nebulae are not objects within our galaxy but galaxies themselves, in motion away from our galaxy. The nearby galaxies he studied recede from us at speeds proportional to their map separation from us (Figure 11).

Before his death in 1953, Hubble made observations with the 200-inch telescope installed on Mount Palomar, California in 1948.

Comment 2. “Distance” and “time”? Look out!
Review Section 2.7, titled *Goodbye “Distance.” Goodbye “Time”*, which first asserted that we cannot apply the concepts of distance and time to our observations of the Universe. The present chapter deeply embodies that assertion.

Determine “distance” with a “standard candle.”

We cannot use laser ranging or classical surveying methods to measure distances outside our galaxy. The most widely used method employs what is called a **standard candle**, a light source whose intrinsic brightness is known. From that intrinsic brightness (more precisely, luminosity) and the apparent brightness (more precisely, flux density) of the object viewed on Earth, we can determine a distance. However, the expanding Universe complicates the analysis, as detailed in Box 4.

Cepheid variables: standard candles

When Hubble did his observations, the major standard candle was one form of the so-called *Cepheid variable* stars. These are stars whose emitted power varies periodically. Their rate of pulsation depends on their emitted power: the longer the pulsation period, the greater the emitted power of the star.

Hubble found Cepheid variable stars in nearby galaxies (but he could not detect them in distant galaxies). To find their approximate distances he classified different galaxies, found the intrinsic brightness of galaxies of a given type that were near enough to allow detection of Cepheid variables they contained, then assumed the same intrinsic brightness for more distant (but still nearby) galaxies of the same type.

Hubble's "island universes" = our galaxies.

Hubble's observations in 1923-1924 showed that most spiral nebulae (for him, fuzzy patches of light in the sky) are much farther away than the limits of our galaxy; they are indeed separate "island universes," or what we now call "galaxies." He also classified "elliptical," "lenticular," and "irregular" galaxies, so-called because of their appearance. All lie outside our own Milky Way galaxy. (Interesting fact: Both "galaxy" and "lactose" come from the Greek and Latin words for milk.) In summary: The Universe extends far beyond our galaxy.

Modern standard candle: Type Ia supernova

Cepheid variable stars are too faint to be seen at distances more than a hundred million light years. For more distant sources, the standard candle of choice is a Type Ia supernova. A Type Ia supernova results when a small, dense white dwarf star gradually accretes mass from a binary companion star, finally reaching a mass at which the white dwarf becomes unstable, collapses, and explodes into a supernova. The "slow fuse" on the gradual accretion process can lead to an explosion of almost the same size on each such occasion, giving us a "standard candle" of the same intrinsic brightness. The brightness of the explosion as seen from Earth provides a measure of the distance to the supernova. The cosmological redshift of light tells us how fast the supernova is receding (Section 14.4). Because supernovae (plural of supernova) are so bright, they can be seen at a very great distance, which brings us information about the Universe most of the way back to the Big Bang.

For astronomers, M and m are magnitudes.

Astronomers plot a quantity called *distance modulus* $m - M$ (also called the *effective magnitude*) where m is the apparent magnitude and M is the absolute magnitude (also called the **intrinsic magnitude**). This difference is related to luminosity distance d_L (Box 6) by the equation

$$m - M = 5\log_{10}\left(\frac{d_\mathrm{L}}{10\,\mathrm{pc}}\right) \qquad (m \text{ and } M \text{ are magnitudes}) \qquad (42)$$

where pc stands for *parsec*, a unit of distance equal to 3.26 light years. Why this peculiar formula? Blame the ancient Greeks, who first quantified the brightness of stars. The key is the realization that M is known (or knowable) for Type Ia supernovae, so measurements of apparent magnitude m, the distance modulus, allow us to solve equation (42) for d_L.

Hubble Diagram

A graph of effective magnitude vs. redshift is called a **Hubble Diagram**. Figure 9 shows the Hubble Diagram for Type Ia supernovae. The thin spread of the curve in the vertical direction confirms that Type Ia supernovae are good standard candles—they all have the same M (when small corrections are applied to raw measurements) so that apparent magnitude m can be used to measure distance.

Expansion speeding up

What are the implications of this analysis? First the obvious: Redshift increases with distance. The next section gives an interpretation of this as a result of cosmological expansion. The more subtle and surprising result is that this expansion is speeding up with t. Chapter 15, Cosmology, elaborates on this second point.

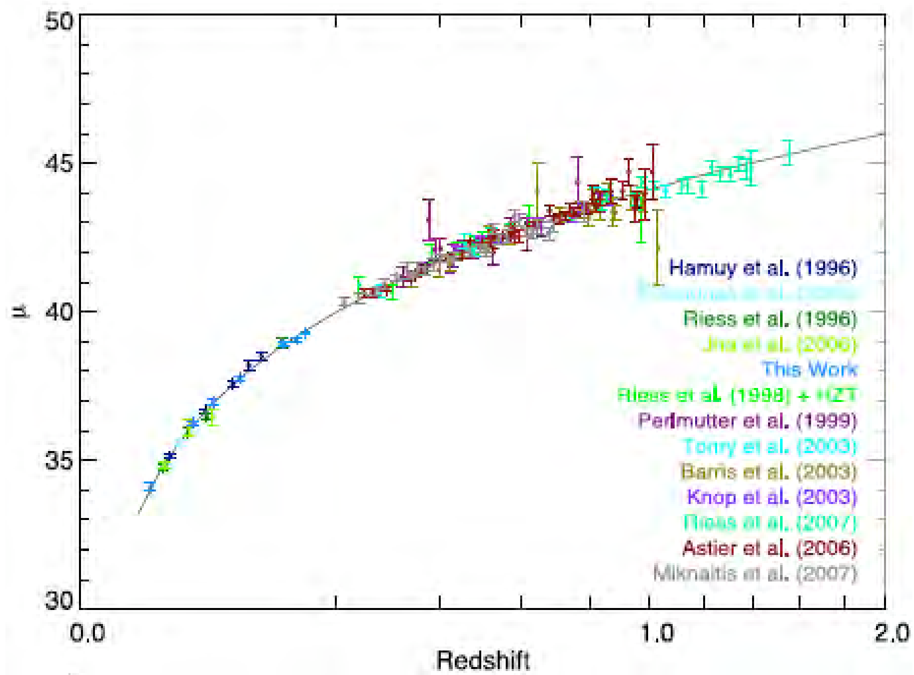


FIGURE 9 Effective magnitude of Type Ia supernovae as a function of their redshift z. The vertical axis is $\mu = m - M$, the difference between apparent magnitude and intrinsic magnitude.

In the future, a second way to measure distances may prove useful in cosmology. From metric (17), objects of known transverse size D at radial coordinate distance χ extend across an angle

$$\theta \approx \frac{D}{S(\chi)R(t_{\text{emit}})} \qquad (|\theta| \ll 1) \tag{43}$$

In flat spacetime the distance would be $d = D/\theta$ if $\theta \ll 1$. In the expanding Universe, cosmologists define the **angular diameter distance** as:

$$d_{\text{A}} \equiv \frac{D}{\theta} = S(\chi)R(t_{\text{emit}}) = \frac{S(\chi)R(t_0)}{1+z} \tag{44}$$

Standard rulers

where we used equation (28). Objects of known transverse size D are called **standard rulers**. Comparing (44) with (52), you can show that $d_{\text{A}} = d_{\text{L}}/(1+z)^2$. Thus, measurements of standard candles and standard rulers for an object of known z yield the same information. The difficulty lies in determining the intrinsic size and luminosities of objects billions of light years away.

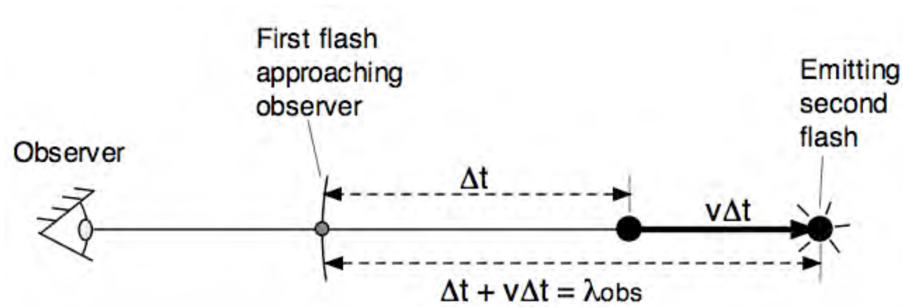


FIGURE 10 Doppler effect observed in a single inertial frame of special relativity, used by Hubble to analyze the speed of receding nearby galaxies.

14.7 ■ LAWS OF RECESSION

Recession rate proportional to "distance"—at least for nearby galaxies.

When Edwin P. Hubble arrived at the Mount Wilson Observatory in California USA in 1919 and began to use the new 100-inch telescope, many astronomers believed that the entire Universe consisted of stars in the Milky Way, what we now call "our galaxy." A disturbing feature of this model of the Universe was the behavior of some of the objects they called **nebulae**. We now know that some nebulae are within our galaxy but most are separate galaxies distant from our own. As early as 1912 Vesto Melvin Slipher had shown that light from many nebulae had significant redshifts, implying that they were moving away from us at high speed. But were these nebulae dim objects in our own galaxy or bright objects outside our galaxy? To answer this question, Hubble needed, first, a relation between redshift and recession velocity. Second, he needed a measure of the distance of these nebulae from us. We examine these tasks in turn.

Velocity vs. Redshift

Hubble used special relativity Doppler shift.

Slipher and Hubble used the Doppler shift of light to find a relation between redshift z and velocity of recession v. They were astronomers, not general relativists. (General relativity theory did not exist when Slipher began his work.) For them the nebulae were speeding away from us in static flat space, and the redshift was a Doppler effect that could be analyzed using special relativity. We will show that this simple analysis gives correct results for nearby nebulae receding from us at relative speeds much less than that of light.

Hubble uses special relativity Doppler shift

Figure 10 introduces the Doppler shift for special relativity. Earlier than the t shown in this figure an object emitted one flash, then moved $v\Delta t$ farther away from the observer, and is emitting the second flash at the instant shown. During that t-lapse the initial flash moved Δt closer to the observer. Let the lapse in t between the two flashes represent one period of a continuous wave. Then the wavelength λ_{obs} detected by the observer has the value shown in the figure. According to Newton, in the rest frame of the source the emitted wavelength would be $\lambda_{\rm source} = \Delta t$. However, we must apply a relativistic correction to Newton's result, because of time stretching.

The t-lapse between flash emissions in the rest frame of the source is different from Δt in the frame of the observer. We say that "the emitting clock runs slow," according to the equation

$$(1-v^2)^{1/2}\Delta t = \Delta t_{\rm source} = \lambda_{\rm source} \qquad \text{(special relativity)} \qquad (45)$$

The ratio of observed wavelength to the wavelength in the frame of the source is

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm source}} = \frac{(1+v)\Delta t}{(1-v^2)^{1/2}\Delta t} = \left(\frac{1+v}{1-v}\right)^{1/2} = 1+z \qquad \text{(special relativity)} \, (46)$$

where we have inserted the definition of redshift z from (28). Nearby galaxies are not moving away from us very fast; for them we may make the approximation:

$$1+z = (1+v)^{1/2}(1-v)^{-1/2} \approx \left(1+\frac{v}{2}\right)^2 \approx 1+v \qquad (v \ll 1) \quad (47)$$

so for slow-moving galaxies the redshift z is equal to the velocity of recession v.

$$v = z \qquad (v \ll 1) \qquad (48)$$

Doppler OK for small z

This Doppler interpretation of the cosmological redshift is valid for $z \ll 1$, because spacetime over such a "small distance" is well approximated by a single flat patch, on which general relativity reduces to special relativity.

Measuring Distance with a "Standard Candle"

Equation (48) gives the velocity of recession. Hubble also needed to know how far away the emitting star is, $\sigma_{\rm now}$. To determine distance we use what is called a **standard candle**, that is, a star whose intrinsic brightness is known. From that intrinsic brightness and the apparent brightness of this star at Earth, one can then determine its distance. However, the expanding Universe complicates this analysis, as detailed in Box 6.

Hubble's Law of Recession

Hubble's law of recession

From the redshift of different galaxies, Hubble now knew from (48) their recession velocities. From the intrinsic brightness of Cephied variable stars and a galaxy of a given type, he could calculate its distance. He found a direct proportion between the average recession velocity of a star and its distance (Figure 11). He called this result the Redshift-Distance Law. We call it **Hubble's Law**, one of the major results of cosmology in the twentieth century:

Box 6. Finding the distance (which distance?) to a standard candle

Consider a star that emits electromagnetic power L (energy per unit time), called **luminosity**, as viewed in its rest frame. We assume that this emission is isotropic, the same in all directions. Place this star at the center of coordinates, $\chi = 0$. Place an observer at a comoving coordinate χ away from the star. In special relativity the power per unit area, also called **flux density** F, reaching an observer at this distant location is:

$$F = \frac{L}{4\pi d^2} \qquad \text{(flat spacetime)} \qquad (49)$$

where d is the distance between star and observer. Now, astronomers cannot measure d directly, so they define a **luminosity distance** $d_{\rm L}$ by the equation

$$d_{\rm L} = \left(\frac{L}{4\pi F}\right)^{1/2} \qquad (50)$$

and report the value of $d_{\rm L}$ for a given star. The luminosity distance $d_{\rm L}$ is the distance from an emitter of power L at which it would produce a flux density F *in flat spacetime*.

In an expanding Universe, F is modified in several ways. First, the metric contains no distance d, but rather a map coodinate χ and an angular factor $S(\chi)$. Second, the energy reaching the observer is reduced by a factor $(1 + z)$ due to the cosmological redshift. Third, the lapse in t that this light takes to arrive at the observer is stretched out by another factor $(1 + z)$. The result is

$$F = \frac{L}{4\pi(1+z)^2 R^2(t_0) S^2(\chi)} \qquad (51)$$

We can measure F and z. Suppose we also know the intrinsic power L of the emitter and, for a specific model of the Universe, the cosmic scale function $R(t_0)$. We can then obtain a measure of the distance from the emitter using (50):

$$S(\chi) = \frac{d_{\rm L}}{(1+z)R(t_0)} \qquad (52)$$

The quantities $d_{\rm L}$ and $S(\chi)$ are measures of distance to our standard candle of luminosity L. You should convince yourself that (50) and (52) taken together imply (51).

$$v = H_0 d_{\rm L} \qquad \text{(nearby galaxy)} \qquad (53)$$

Hubble constant H_0

Here H_0 is called the **Hubble constant** and refers to its value at the present age of the Universe. The current value of the Hubble constant in units used by astronomers is

$$H_0 = 73 \pm 2\ \frac{\text{kilometer/second}}{\text{Megaparsec}} \qquad (54)$$

where one Megaparsec equals 3.26 million light years. Expressed in geometric units, this has the value:

$$H_0 = (8.0 \pm 0.2) \times 10^{-27}\ \text{meter}^{-1} \qquad (55)$$

Robertson-Walker Law of Recession

Recession at great distance and great speed

What happens when we do not make the assumption that emitting galaxies are nearby? We use the Robertson-Walker metric to answer this question. Write the spacelike form of (17) for fixed ϕ-coordinate.

$$d\sigma^2 = R^2(t)d\chi^2 - dt^2 = ds^2 - dt^2 \qquad (d\phi = 0) \qquad (56)$$

At fixed t_1 this equation can be integrated to give the distance d:

$$d_1 = R(t_1)\chi \qquad (dt = 0) \qquad (57)$$

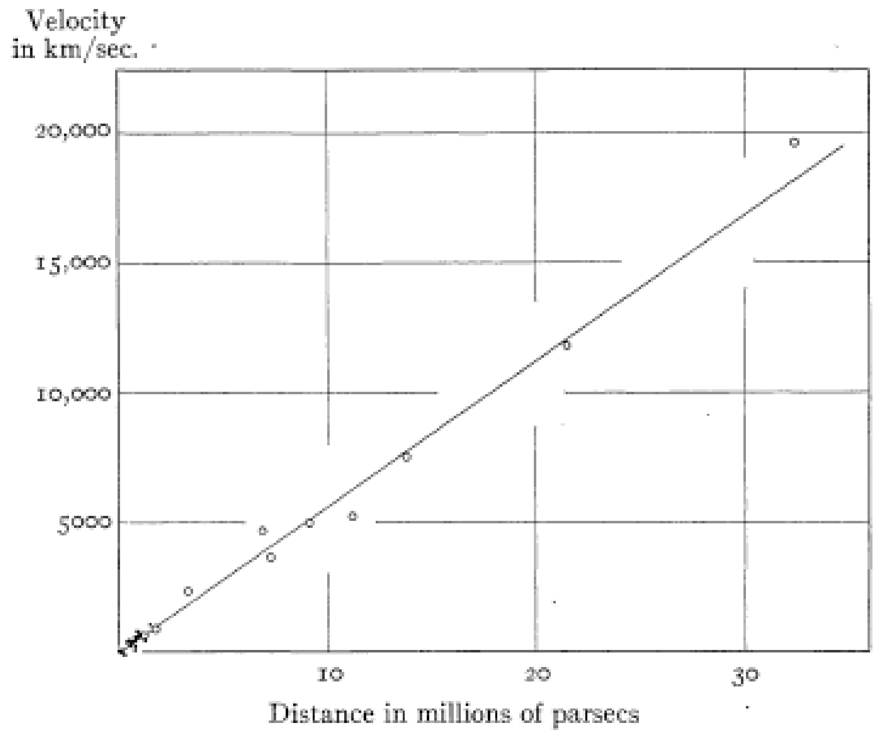


FIGURE 11 A plot of recession velocity as a function of distance by Hubble and Milton Humason (1931). Open circles represent averages of groups of galaxies; solid dots near the origin show individual galaxies from an earlier paper by Hubble. A parsec equals 3.26 light-years, so the most distant group of galaxies is approximately 100 million light-years distant—"nearby" by modern standards. The Hubble constant derived from the slope of the line in this figure is different from the current value, equation (54); see the exercises.

Assume that a distant galaxy is at rest in comoving coordinates χ (and ϕ), so that χ remains constant. Then at a later t_2, the galaxy is at distance

$$d_2 = R(t_2)\chi \qquad (dt = 0) \tag{58}$$

The recession speed at t is expressed using elementary calculus:

$$\begin{aligned} v_\mathrm{r} &= \lim_{t_2 \to t_1} \frac{d_2 - d_1}{t_2 - t_1} = \lim_{t_2 \to t_1} \frac{R(t_2) - R(t_1)}{t_2 - t_1}\chi \\ &\equiv \dot{R}\chi = \left(\frac{\dot{R}}{R}\right) R\chi \equiv H(t)d \end{aligned} \tag{59}$$

Hubble parameter

where the **Hubble parameter** $H(t)$ is defined as

$$H(t) \equiv \frac{\dot{R}(t)}{R(t)} \qquad \text{(Hubble parameter)} \tag{60}$$

We can expect the Hubble parameter to have different values at different t-values during the evolution of the Universe. Its current value is given the symbol $H_0 \equiv H(t_0)$.

As noted in Section 14.6, astronomers cannot measure d directly. Instead they measure $d_{\rm L}$ or $d_{\rm A}$. When either of these is plotted against redshift z, the resulting relation is linear only for $z \ll 1$. At high redshift the behavior depends on the detailed form of the scale function $R(t)$.

We need radial function $R(t)$.

We have milked about as much information out of the Robertson-Walker metric as we can without knowing the t-development of the scale function $R(t)$, which derives from the constituents of the Universe as it expands. The following Chapter 15, Cosmology, develops this scale function from a combination of observed redshifts (28) using standard candles at different distances and further solutions of Einstein's equations. The result provides our current picture of the history of the Universe and gives us insight into its possible futures.

14.8 ■ EXERCISES

1. Tangential Momentum

Carry out the full derivation of the tangential momentum Q_ϕ in equation (41), including equations similar to (32) through (38) and a figure similar to Figure 7.

2. Energy not a Constant of Motion

Show that a derivation of the energy as a constant of motion is not possible. Begin by varying only the t-value of the central event in Figure 7. What derails this derivation, making it impossible to complete?

3. Transverse Motion

A galaxy is five billion light-years distant. The most sensitive microwave array can detect a displacement angle as small as 50 microarcseconds transverse to the radial direction of sight. (One second of arc is 1/3600 of a degree.) With what transverse speed, as a fraction (or multiple) of the speed of light, must the distant source move in order that its transverse motion be detected in a 100-year human lifetime? Assume the Universe is flat.

5. Hubble's Error

Compare the value of the slope in Figure 11 with the modern value of Hubble's constant given in equations (54) and (55). By what factor was Hubble's result different from the current value of the Hubble constant?

6. 'Distance' and 'velocity' in Hubble's Law

Section 14.7 states that Hubble *found a direct proportion between the average recession velocity of a star and its distance,* which violates our rule to avoid words like *distance* when we describe observations in curved spacetime.

A. Review Section 14.7 and explain why the word *distance* does not have a unique meaning in this case.
B. Explain why the word *velocity* does not have a unique meaning.
C. Does the relative velocity of two *distant* objects have a unique meaning in curved spacetime? in flat spacetime?
D. Rewrite the Section 14.7 statement of Item A to avoid difficulties of words like *velocity* and *distance.*

14.9 ■ REFERENCES

Richard Dawkins quoted in *A Universe from Nothing* by Lawrence M. Krauss, New York, Atria Paperback, 2012, page 187.

Final cartoon by Jack Ziegler in the New Yorker Magazine July 13, 1998. IF we use it, we need formal permission.

Figure 1 from:
`http://thinkexist.com/quotes/like/once_you_can_accept_the_universe_as_being/33`

Figure 4 from From the Sloan Digital Sky Survey:
`http://www.sdss.org/includes/sideimages/quasar_stack.html`

Picture of Edwin Hubble from the cover of Time Magazine, February 9, 1948

Figure 11 from "The Velocity-Distance Relation Among Extra-Galactic Nebulae," by Edwin Hubble and Milton L. Humason, Astrophysical Journal, Volume 74, 1931, pages 43 to 80.

For a history of non-Euclean geometry, see Steven Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, New York, John Wiley & Sons, 1972, Chapter 1.

For an account of Hubble's discovery of distant galaxies and Hubble's Law, see *Edwin Hubble, Mariner of the Nebulae* by Gale E. Christianson, 1995 University of Chicago Press and *Edwin Hubble, The Discoverer of the Big Bang Universe* by Alexander S. Sharov and Igor D. Novikov, translated by Vitaly Kisin, 1993 Cambridge University Press.

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