Chapter 21. Inside the Spinning Black Hole

- *Why can't I escape from inside the event horizon of the* non-spinning *black hole?*
- *How can I escape from inside the spinning black hole?*
- *When I do emerge from inside the spinning black hole, where am I?*
- *After I emerge from inside the spinning black hole, can I return to Earth?*
- *What limits does my finite wristwatch lifetime place on my personal exploration of spacetimes?*
- *What limits are there on spacetimes that a group in a rocket can visit?*

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CHAPTER

21 Inside the Spinning Black Hole

Edmund Bertschinger & Edwin F. Taylor *

The non-spinning black hole is like the spinning black hole, but with its gate to other Universes closed. For the spinning black hole, the gate is ajar.

—Luc Longtin

21.1 ■ ESCAPE FROM THE BLACK HOLE

Exit our Universe; appear in a "remote" Universe!

Travel to another Universe . . .

. . . on a one-way ticket!

Begin with effective potential.

Chapters 18 through 20 examined orbits of stones and light around the spinning black hole. We study orbits to answer the question, "Where *do* we go near the spinning black hole?" The present chapter shifts from orbits to topology—the connectedness of spacetime. Topology answers the question, "Where *can* we go near the spinning black hole?" *Astonishing result:* We can travel from our Universe to other Universes. These other Universes are "remote" from ours in the sense that from them we can no longer communicate with an observer in our original Universe, nor can an observer in our original Universe communicate with us. *Worse:* Once we leave our Universe, we cannot return to it. Sigh!

Figure 1 previews this chapter by examining the r-motion of a free-fall stone—or observer—in the effective potential of the spinning black hole. Free stones with different map energies have different fates as they approach the spinning black hole from far away. Two stones with map energies $(E/m)_2$ and $(E/m)_3$, for example, enter unstable circular orbits. In contrast, the stone with map energy $(E/m)_4$ reaches a turning point where its map energy equals the effective potential, then it reflects outward again into distant flat spacetime. *Question:* What happens to a stone with map energy $(E/m)_1$? Two question marks label its intersection with the forbidden region inside the Cauchy horizon. Does the stone reflect from this forbidden region? Does it move outward again through the Cauchy and event horizons? Does it emerge into our Universe? into some other Universe? The present chapter marshalls general relativity to answer these questions.

*Draft of Second Edition of *Exploring Black Holes: Introduction to General Relativity*

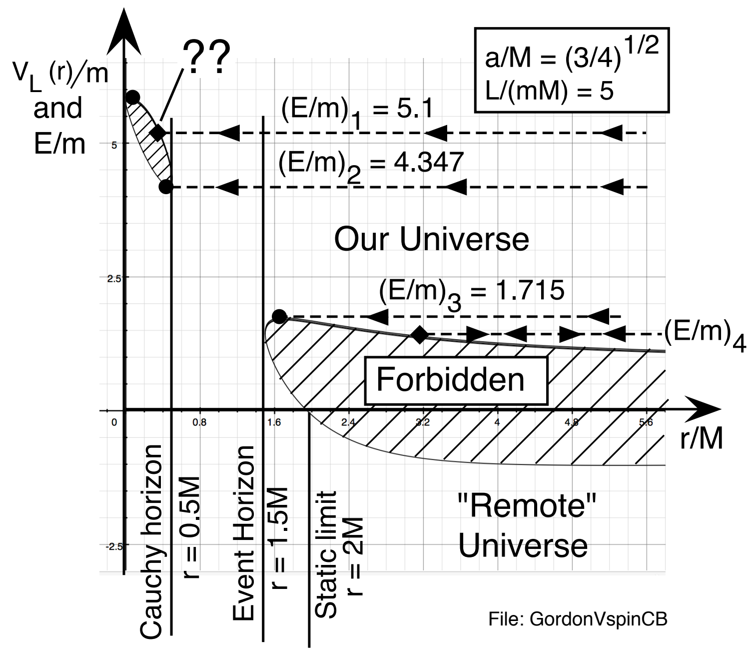

FIGURE 1 Effective potential for a stone with $L/(mM) = 5$ near a spinning black hole with $a/M = (3/4)^{1/2}$. What happens at the intersection of the horizontal line $(E/m)_1$ with the forbidden region inside the Cauchy horizon? (Adapted from Figure 5 in Section 18.4.)

Carter-Penrose diagram

The idea of traveling from our Universe to another Universe is not new. In 1964 Roger Penrose devised, and in 1966 Brandon Carter improved, what we now call the **Carter-Penrose diagram** for spacetime, a navigational tool for finding one's way across Universes. This diagram will be the subject of the following sections.

21.2 ■ THE CARTER-PENROSE DIAGRAM FOR FLAT SPACETIME

Begin around the edges, then fill in.

Global metric flat spacetime

As usual, we develop our skills gradually, first with flat spacetime, then with the non-spinning black hole, and finally with the spinning black hole. Here is a global metric on an $[x, t]$ slice in flat spacetime:

$$d\tau^2 = dt^2 - dx^2 \qquad \text{(global metric, flat spacetime)} \qquad (1)$$

$$-\infty < t < \infty, \quad -\infty < x < \infty \qquad (2)$$

The following transformation from $[t, x]$ to $[v, u]$ corrals the infinities in (2) onto a single flat page:

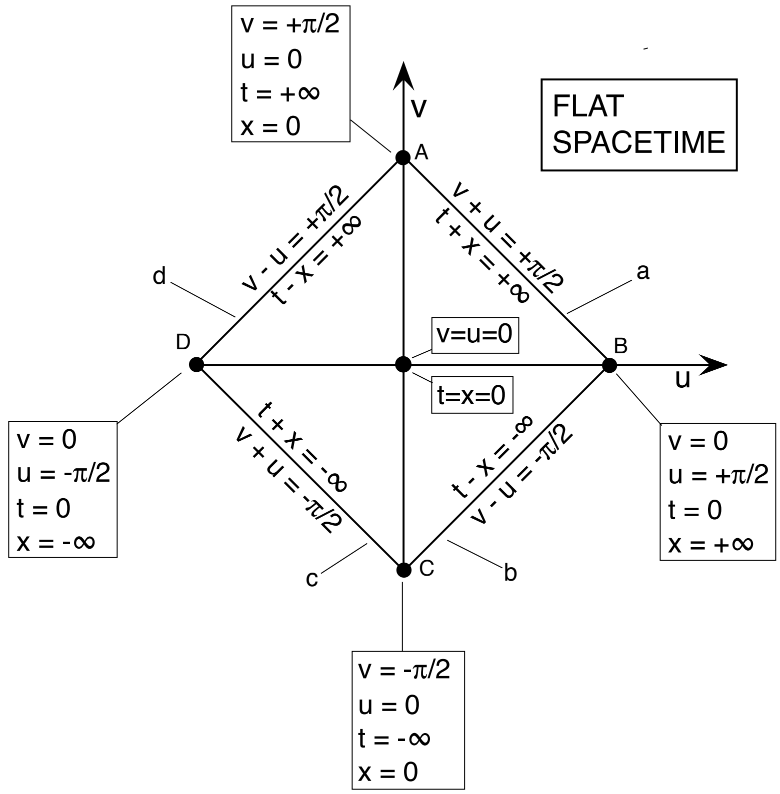

FIGURE 2 Points and lines on the boundaries in the Carter-Penrose diagram for flat spacetime.

$$t = \frac{1}{2}\left[\tan(u+v) - \tan(u-v)\right] \quad \text{(global coordinates, flat spacetime)} \qquad (3)$$

$$x = \frac{1}{2}\left[\tan(u+v) + \tan(u-v)\right] \qquad (4)$$

$$-\pi/2 < v < +\pi/2, \quad -\pi/2 < u < +\pi/2 \qquad (5)$$

QUERY 1. Coordinate ranges
Show that transformations (3) and (4) convert the coordinate ranges of t and x in (2) into the coordinate ranges of v and u in (5). In other words, the Carter-Penrose diagram brings map coordinate infinities onto a finite diagram.

Carter-Penrose diagram

Figure 2 shows the result of this transformation, which we call the **Carter-Penrose diagram**. It plots positive infinite t at point A, negative infinite t at point C, distant positive x at point B, and distant negative x at

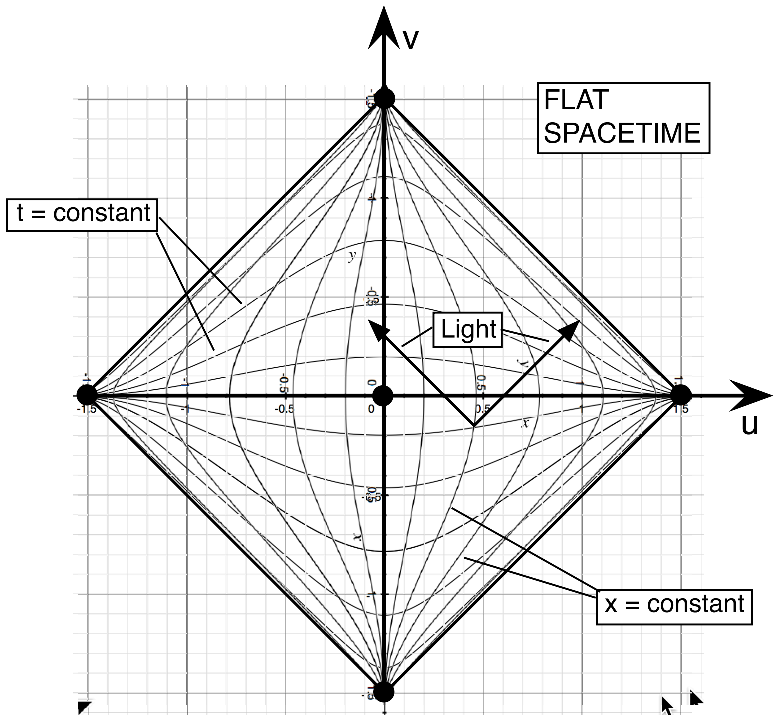

FIGURE 3 The Carter-Penrose diagram that fills in coordinates of Figure 2 on the $[x, t]$ slice of flat spacetime. These curves plot v vs. u from the inverse of equations (3) through (5). These particular conformal coordinates preserve the $\pm 45^o$ angles for worldlines of light.

point D. In Query 2 you use equations (3) through (5) to verify map coordinate values in this figure.

QUERY 2. Points and boundaries in the Carter-Penrose diagram

Use equations (3) and (4) to verify the following statements about points A through D and boundaries a through d in Figure 2:

A. Show that when $u = 0$ then $x = 0$, and when $v = 0$ then $t = 0$.

B. Verify the boxed values of t and x at points A through D.

C. Verify the values of $v + u$ along the two lines labeled a and c.

D. Verify the values of $v - u$ along the two lines labeled b and d..

E. Verify the values of $t + x$ along the two lines labeled a and c.

F. Verify the values of $t - x$ along the two lines labeled b and d.

The Carter-Penrose diagram is a **conformal diagram** that brings global coordinate infinities onto the page. A conformal diagram is simply an ordinary spacetime diagram for a metric on which we have performed a particularly

Conformal diagram

clever coordinate transformation. This particular coordinate transformation preserves the causal structure of spacetime defined by the light cone.

To find the global metric on the $[u, v]$ slice for flat spacetime, take differentials of (3) and (4) and rearrange the results:

$$dx = \frac{1}{2}\left[\frac{du + dv}{\cos^2(u+v)} + \frac{du - dv}{\cos^2(u-v)}\right] \tag{6}$$

$$dt = \frac{1}{2}\left[\frac{du + dv}{\cos^2(u+v)} - \frac{du - dv}{\cos^2(u-v)}\right] \tag{7}$$

$$-\pi/2 < v < +\pi/2, \quad -\pi/2 < u < +\pi/2 \tag{8}$$

Global metric in u, v coordinates

Substitute dx and dt from (6) and (7) into global metric (1) and collect terms. Considerable manipulation leads to the global metric on the $[u, v]$ slice:

$$d\tau^2 = \frac{dv^2 - du^2}{\cos^2(u+v)\cos^2(u-v)} \tag{9}$$

$$-\pi/2 < v < +\pi/2 \qquad -\pi/2 < u < +\pi/2 \tag{10}$$

Equation (9) has the same form as equation (1) except it is multiplied by $[\cos^2(u+v)\cos^2(u-v)]^{-1}$, called the **conformal factor**. Indeed, equations (9) and (10) are examples of a **conformal transformation:**

DEFINITION 1. Conformal transformation

Definiton: **Conformal transformation**

A conformal transformation has two properties:

- It transforms global coordinates.
- The new global metric that results has the same form as the old global metric, multiplied by the *conformal factor.*

Conformal factor

The transformation (3) through (5) has both of these properties. In particular, the resulting metric (9) has the same form (a simple difference of squares) as (1), multiplied by the conformal factor $[\cos^2(u+v)\cos^2(u-v)]^{-1}$.

Infinities on the $[x, t]$ slice correspond to finite (non-infinite) values on the $[u, v]$ slice, due to the conformal factor in (9), which goes to $x + t = \pm\infty$ or $x - t = \pm\infty$ when $u + v = \pm\pi/2$ or $u - v = \pm\pi/2$, as shown around the boundaries of Figure 2.

For the motion of light, set $d\tau = 0$ in (9). Then the numerator $dv^2 - du^2 = 0$ on the right side ensures that $dv = \pm du$, so the worldline of light remains at $\pm 45°$ on the $[u, v]$ slice. Therefore a light cone on the $[u, v]$ slice has the same orientation as on the $[x, t]$ slice. We deliberately choose conformal coordinates to make this the case.

Worldlines of light at $\pm 45°$

QUERY 3. Standing still; limits on worldlines

A. Show that when $dx = 0$ in (6), then $du = -du$, which means that $du = 0$. *Result:* The stone with a vertical worldline on the $[x, t]$ slice has a vertical worldline on the $[u, v]$ slice.

B. Show that in Figure 2 the worldline of every stone lies inside the light cone $\pm 45°$.

QUERY 4. You cannot "reach infinity."

Show that as $x \to \pm\infty$ the global equation of motion dx/dt for a stone takes the form $dx/dt \to \pm 0$. Therefore a stone cannot reach that limit, any more than it (or you!) can reach infinity.

? **Objection 1.** *Are these predictions real? They sound like science fiction to me!*

! We do not use the word "real" in this book; see the Glossary. These predictions can in principle be validated by future observations carried out by our distant descendents. In that sense they are scientific. They also satisfy *Wheeler's radical conservativism:* "Follow what the equations tell us, no matter how strange the results, then develop a new intuition."

21.3 ■ TOPOLOGY OF THE NON-SPINNING BLACK HOLE

The one-way worldline

We move on from flat spacetime to spacetime around the non-spinning black hole. Equations (17) and (18) of Section 8.4 connect the global r-motion of a stone to the effective potential $V_L(r)$:

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{E}{m}\right)^2 - \left(\frac{V_L(r)}{m}\right)^2 \qquad (11)$$

A remote Universe

Because all terms in this equation are squared, the effective potential $V_L(r)$ and the map energy E/m can be either positive or negative, as shown in Figure 4. our Universe lies above the forbidden region. Below the forbidden region lies a second, "remote" Universe.

Meaning of "forbidden"

What does "forbidden" mean? Equation (11) tells us that global r-motion $dr/d\tau$ becomes imaginary when $(E/m)^2$ is smaller than $(V_L/m)^2$. In other words, neither stone nor observer can exist inside the forbidden region.

The forbidden region prevents the direct passage from our Universe to this remote Universe. To do so we would have to move inward through the event horizon with positive map energy, then use rocket blasts to re-emerge below

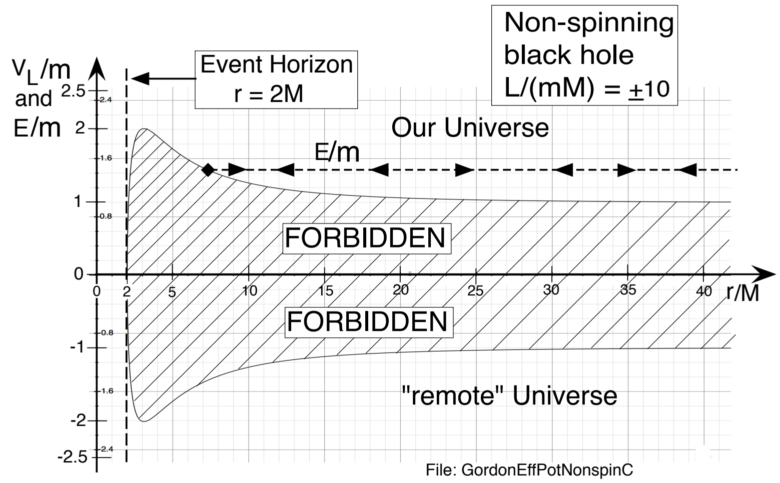

FIGURE 4 Effective potential for the non-spinning black hole, copy of Figure 5 of Section 8.4.

Door to remote Universe is closed.

the forbidden region with negative map energy. But inside the event horizon motion to smaller r is inevitable. *Result:* For the non-spinning black hole the door to to the remote Universe is closed.

Figure 5 displays the double-ended funnel-topology of the non-spinning black hole. The upper and lower flat surfaces represent flat spacetime in our Universe and in the remote Universe, respectively. The pinched connection in

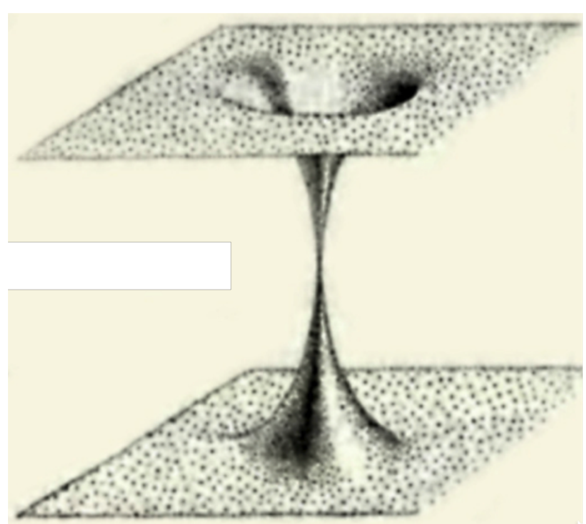

FIGURE 5 Topology of the non-spinning black hole that supplements Figure 4. The upper flat surface represents our Universe. It is connected to a remote Universe (lower flay surface) by the impassable *Einstein-Rosen bridge.*

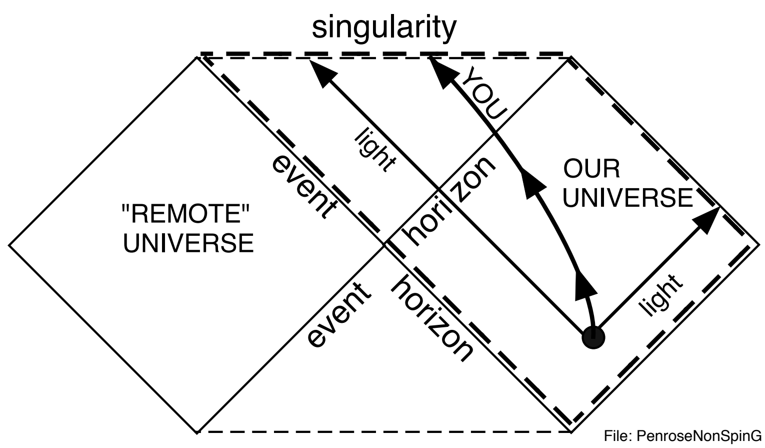

FIGURE 6 Carter-Penrose diagram for the non-spinning black hole, which has two event horizons. Heavy dashed lines enclose spacetime spanned by the Schwarzschild Metric, which has access to only one of these event horizons. From our Universe a stone, light flash, or observer cannot reach the "Remote" Universe in Figures 6 and 1 by crossing the second event horizon.

Einstein-Rosen bridge unpassable

the center, called the **Einstein-Rosen Bridge,** is too narrow for a stone or light flash to pass between the two Universes.

Now turn attention to the Carter-Penrose diagram for the non-spinning black hole, displayed in Figure 6. This two-dimensional diagram suppresses the ϕ-coordinate, leaving t and r global coordinates. The Schwarzschild metric, equation (5) in Section 3.1, becomes:

$$d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 \tag{12}$$

$$-\infty < t < +\infty, \qquad 0 < r < \infty \tag{13}$$

Two event horizons

In this Carter-Penrose diagram an inward-moving stone or light flash crosses the event horizon, then moves inevitably to the singularity represented by the spacelike horizontal line. Topologically there is a second event horizon that is not available to this stone or light flash, because their worldlines are corralled within the upward-opening light cones.

21.4 ■ TOPOLOGY OF THE SPINNING BLACK HOLE

No two-way worldline!

Reflect outward from inside Cauchy horizon?

Figure 1 displays the effective potential for a stone with map angular momentum $L/(mM) = 5$ near the spinning black hole with $a/M = (3/4)^{1/2}$. The striking new feature of this effective potential is the added forbidden region *inside* the Cauchy horizon. This added forbidden region raises the

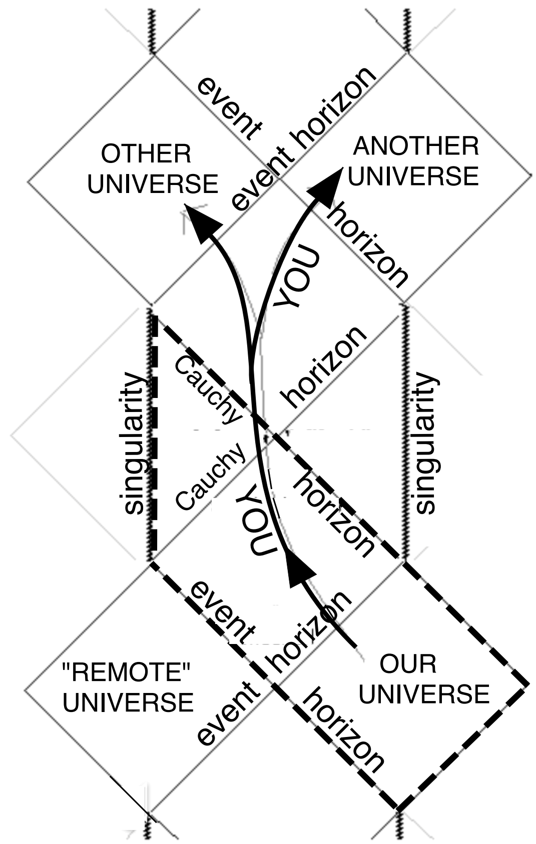

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FIGURE 7 Carter-Penrose diagram of the spinning black hole that answers questions posed in the caption to Figure 1. The heavy dashed line shows the boundaries of Doran global coordinates, which enclose one event horizon and one Cauchy horizon. With calibrated rocket blasts, you can choose to enter either the Other Universe or Another Universe at the top of the diagram. The upward orientation of your worldline shows that you cannot return to our Universe once you leave it—according to general relativity.

possibility that the stone with, say, $(E/m)_1 = 5.1$ can reflect from this forbidden region and move back outward into a distant region of flat spacetime.

Worldline moves between Universes.

Figures 7 and 8 present the topology of such a spinning black hole. You, the observer who travels along the worldline in Figure 7, start in our Universe, pass inward through the event horizon and the Cauchy horizon, reflect from the forbidden region inside the Cauchy horizon, and emerge from a second Cauchy horizon. Then, with the use of rockets, you can choose which event horizon to cross into one of two alternative Universes at the top of this diagram.

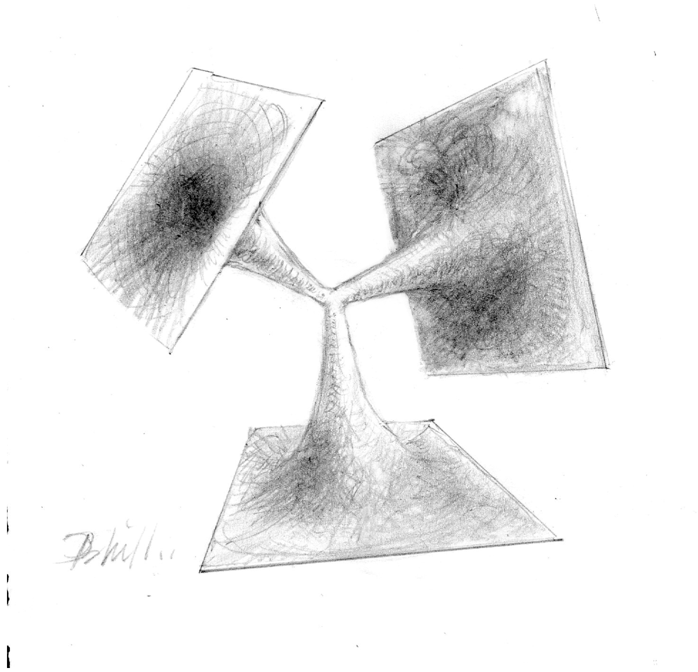

FIGURE 8 Topology of spacetime around the spinning black hole. In this case the central Einstein-Rosen bridge is wide enough for a traveler to pass through on her one-way trip to another Universe. Indeed, she may use rocket thrusts to choose between two alternative Universes. This figure supplements Figures 1 and 7.

To construct Figure 7 suppress the Φ-coordinate of the Doran metric, equation (4) in Section 17.2. The result:

$$d\tau^2 = dT^2 - \left[\left(\frac{r^2}{r^2+a^2}\right)^{1/2} dr + \left(\frac{2M}{r}\right)^{1/2} dT\right]^2 \quad \text{(Doran, } d\Phi = 0) \quad (14)$$

$$-\infty < T < \infty, \quad 0 < r < \infty$$

?

Objection 2. *Why are the lines labeled "singularity" in Figure 7 vertical, while the line labeled "singularity" in Figure 6 is horizontal?*

!

These diagrams show *topology*: where you *can* go, and where you *cannot* go. Categories "vertical" and "horizontal" in such a diagram carry no prediction for observation. Each case shows that you cannot climb out of the singularity.

Emerge into another Universe

The heavy dashed line in Figure 7 outlines the spacetime region included in Doran global coordinates. Notice that this included region is only part of available spacetime. Compare the worldline in Figure 7 with the horizontal line $(E/m)_1$ in Figure 1. This comparison shows that the reflected observer does not re-emerges into our Universe, but into one of the alternative Universes at the top of Figure 7. *Conclusion:* For the spinning black hole, the gate between alternative Universes is ajar (initial quote of this chapter). But your worldline in Figure 7 moves relentless upward; you cannot return to the Universe you have left. You can't go home again!

?

Objection 3. *How can I tell that I have reached the limits of a map, but not the limits of spacetime, when there is uncharted territory ahead.*

!

If you reach the boundary of a global coordinate system in finite wristwatch time (so that $dT/d\tau \to \infty$) or some other singularity arises, and if you have not reached a singularity, then you have just demonstrated that the original coordinates are incomplete and need to be extended. This limited feature of global coordinates is called **geodesic incompleteness:** There exist (portions of) geodesics (or other curves) that reach the edge of your coordinate range and continue beyond your present global coordinates—unless you introduce new global coordinates that cover the new range.

Geodesic incompleteness

?

Objection 4. *How many different Universes are there?*

!

In principle the Carter-Penrose diagram of Figure 7 extends indefinitely both upward and downward, embracing an unlimited number of Universes.

Triple funnel for spinning black hole

Figure 8 displays the topology through which you pass along the worldline of Figure 7. You enter the funnel from Our Universe, then use rockets to choose the Universe into which you emerge. Your worldline in Figure 7 shows that you cannot re-enter that funnel in order to return to our Universe. Your trip between Universes is a one-way street!

?

Objection 5. *So WHERE are these other Universes? Show them to me!*

!

Spacetimes multiply inside the event horizon of the spinning black hole. What does this mean for regions far from the spinning black hole? *Big surprise:* An observer can use rockets to maneuver inside the event horizon of the spinning black hole in order to choose the remote Universe into which she emerges. *Example:* Figure 7 shows that the "bouncing" traveler with $(E/m)_1$ in Figure 1 can emerge into either one of two alternative Universes shown in Figure 8. *Conclusion:* Neither of these alternative Universes "exist" in our Universe in the everyday sense—but you can travel there!

21.5 ■ EXERCISES

SUGGESTED EXERCISES, PLEASE!

21.6 ■ REFERENCES

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Kruskal, M.D, “Maximal Extension of Schwarzschild Metric,” Physical Review, Volume 119, Number 1, September 1, 1960, pages 1743-1745.

Figure 5 adapted from From Kip Thorne, *Black Holes and Time Warps: Einstein’s Outrageous Legacy,* New York, W. W. Norton, 1994, page 487.

Figure 8 drawn by Thomas Dahill.

Useful Carter-Penrose diagrams reference: William J. Kaufmann, III, *Black Holes and Warped Spacetime*, W.H. Freeman Co., San Francisco, 1979.

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